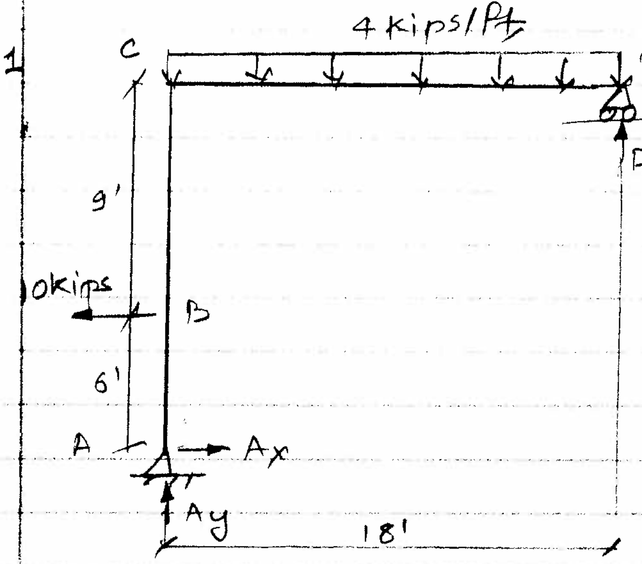
Exam-2 solutions

Q.1



$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow A_x - 10 = 0$

$A_x = 10$ kips $(\rightarrow)$

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow$

$-10 \times 6 + (4 \times 18) \times 9 - D_y(18) = 0$

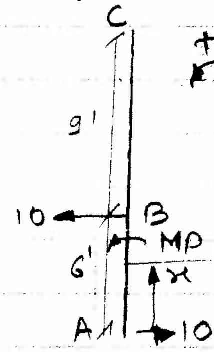
$D_y = 32.67$ kips $(\uparrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y + D_y - 4(18) = 0$

$A_y = 39.33$ kips $(\uparrow)$

(Ignore axial deformation).

P system (Anticlockwise +ve)



From A to B $(0 \le x \le 6)$

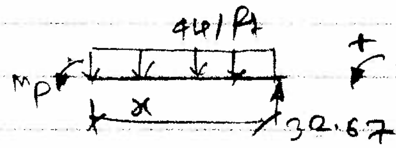
$\overset{+}{\circlearrowleft} \Sigma M = 0 = 10x + M_p$

$M_p = -10x$

From B to C $(6 \le x \le 15)$

$\overset{+}{\circlearrowleft} \Sigma M = 0 = 10x - 10(x-6) + M_p$

$M_p = -60$

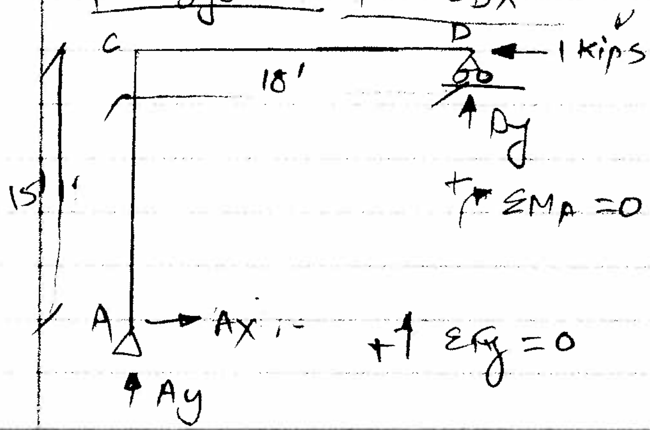


$(0 \le x < 18)$

$\overset{+}{\circlearrowleft} \Sigma M = 0 = M_p + 32.67x - \frac{4x^2}{2}$

$M_p = -32.67x + 2x^2$

Q system for δ_{DX}



$\Sigma F_x = 0 \Rightarrow A_x - 1 = 0$

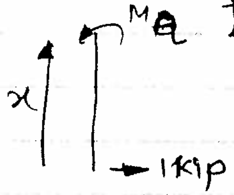
$A_x = 1$ kips $(\rightarrow)$

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow -D_y(18) - 1(15) = 0$

$D_y = -\frac{15}{18} = -\frac{5}{6}$ kips $(\downarrow)$

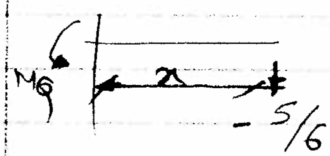
$+\uparrow \Sigma F_y = 0 \quad D_y + A_y = 0 \Rightarrow A_y = \frac{5}{6}$ kips $(\uparrow)$

For AB & BC



$\overset{+}{\curvearrowleft} \Sigma M = 0 = 1(x) + M_p = 0 \rightarrow M_p = -x$

for CD ($0 \le x \le 18$)



$\overset{+}{\curvearrowleft} \Sigma M = 0 \Rightarrow M_Q - \left(\frac{5}{6}\right)x = 0$

$M_Q = \frac{5}{6}x$

$$\Sigma Q \cdot \delta_{Dx} = \Sigma \int \frac{M_Q M_p}{EI}\, dx$$

$$1(\delta_{Dx}) = \int_0^6 \frac{(-10x)(-x)}{EI}\, dx + \int_6^{15} \frac{(-60)(-x)}{EI}\, dx + \int_0^{18} \frac{(-32.67x + 2x^2)}{EI}\left(\frac{5}{6}x\right) dx$$

$$= \int_0^6 \frac{10x^2}{EI}\, dx + \int_6^{15} \frac{60x}{EI} - \int_0^{18} \frac{27.23x^2}{EI} + \int_0^{18} \frac{1.67x^3\, dx}{EI}$$

$$= \frac{1}{EI}\left\{\left[\frac{10x^3}{3}\right]_0^6 + \left[\frac{\overset{30}{\cancel{60}}x^2}{\cancel{2}}\right]_6^{15} - \left[\frac{27.23x^3}{3}\right]_0^{18} + \left[\frac{1.67x^4}{4}\right]_0^{18}\right\}$$

$$= \frac{1}{EI}\left[720 + 5670 - 52935.12 + 43827.48\right]$$

$$\delta_{Dx} = \frac{-2717.64}{EI} \quad (\rightarrow)$$

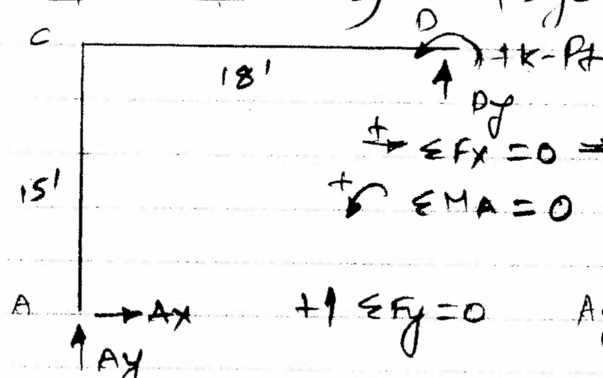
with including axial deformation.

$$\Sigma Q\, \delta_{Dx} = \Sigma \frac{M_Q M_p\, dx}{EI} + \Sigma \frac{F_Q F_p L}{AE}$$

$$= \frac{-2717.64}{EI} - \frac{\left(5/6\right)(32.67) \times 15}{AE}$$

$$\delta_{Dx} = \frac{-2717.64}{EI} - \frac{408.38}{AE} \quad (\rightarrow)$$

Slope at D. (θ_D) → φ system



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow A_x = 0$

$\overset{+}{\curvearrowleft} \Sigma M_A = 0 \quad 1 + D_y(18) = 0$

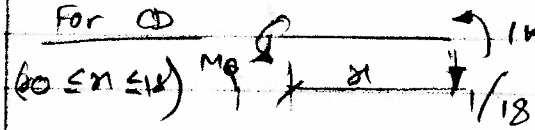
$D_y = -1/18$ kips (↓)

$+\uparrow \Sigma F_y = 0 \quad A_y + D_y = 0$

$A_y = 1/18$ kips (↑)

For AC $\quad M_\phi = 0$

For CD



$\overset{+}{\curvearrowleft} \Sigma M = 0 = M_\phi + 1 - (1/18)x = 0$

$M_\phi = \left(\frac{x}{18}\right) - 1$

$$\int \phi\, \theta_D = \int \frac{M_Q M_\phi}{EI}\, dx$$

$$= \frac{1}{EI}\int_0^{18} \left(-32.67x + 2x^2\right)\left(\frac{x}{18} - 1\right) dx$$

$$= \frac{1}{EI}\int_0^{18} \left[\frac{-32.67x^2}{18} + 32.67x + \frac{2x^3}{18_9} - 2x^2\right] dx$$

$$= \frac{1}{EI}\left[\frac{-1.815x^3}{3} + \frac{32.67x^2}{2} + \frac{x^4}{36} - \frac{2x^3}{3}\right]_0^{18}$$

$$= \frac{1}{EI}\left[-3528.36 + 5292.54 + 2916 - 3888\right]$$

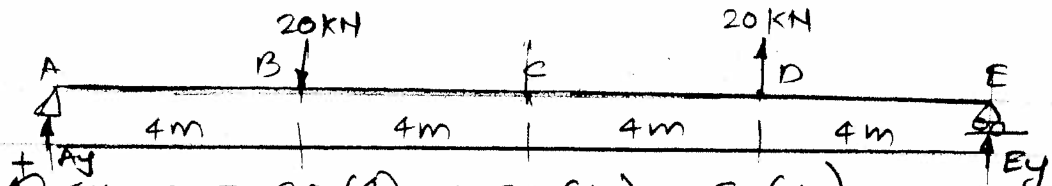
$$\boxed{\theta_D = \frac{792.18}{EI}} \quad \curvearrowleft \text{ radians.}$$

RISA check → $I = 100\ in^4$, $E = 29000$ ksi, $A = 10\ in^2$

$$\delta_{Dx} = \frac{-2717.64\,(1728)}{29000\,(100)} - \frac{408.38 \times 12}{10 \times 29000} = 1.64\text{in } (\rightarrow)$$

$$\theta_D = \frac{792.18\,(144)}{29000\,(100)} = 0.039 \text{ radians } (\curvearrowleft)$$

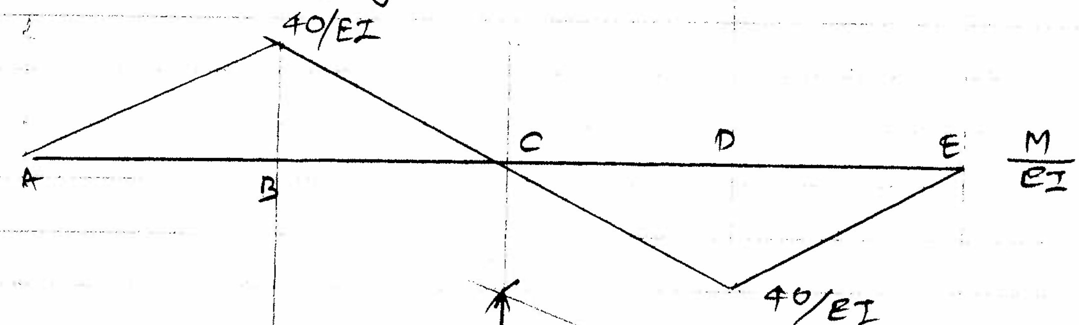
Q.2)

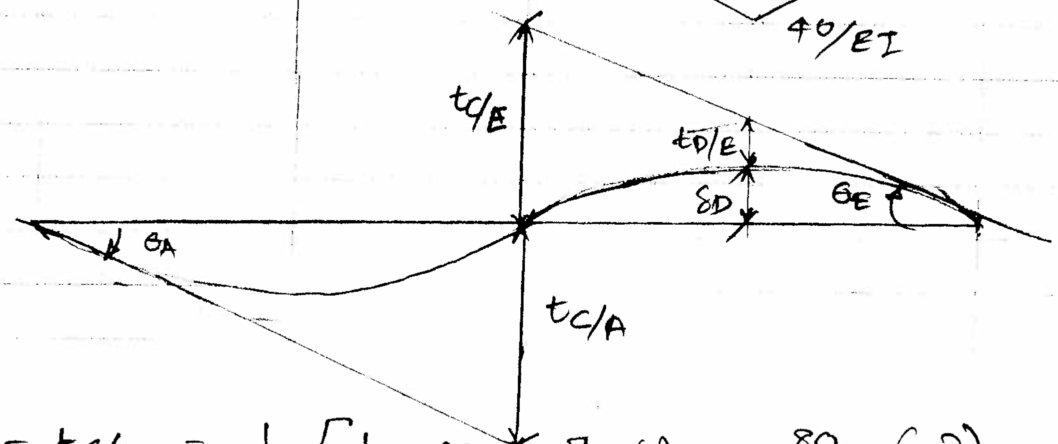


$+\circlearrowleft \Sigma M_A = 0 = -20(4) + 20(12) + E_y(16) =$

$E_y = 10 \text{ kN} (\downarrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y = 10 \text{ kN} (\uparrow)$





$$\theta_A = \frac{t_{C/A}}{8} = \frac{1}{8}\left[\frac{1}{2}\times\frac{40}{EI}\times 8\right]\times(4) = \frac{80}{EI} \ (\circlearrowright)$$

By symmetry, $\theta_A = \theta_E \Rightarrow t_{C/A} = t_{C/E} = \frac{640}{EI}$

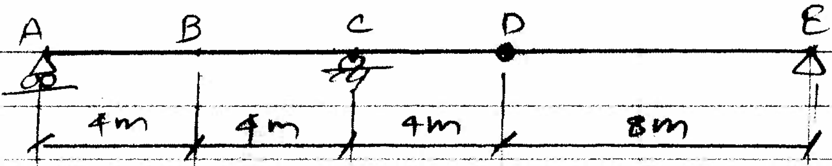
$$\delta_D = 4\theta_E - t_{D/E}$$

$$= 4\left(\frac{80}{EI}\right) - \left[\frac{1}{2}\times\frac{40}{EI}\times 4\right]\times\left(\frac{1}{3}\times 4\right)$$

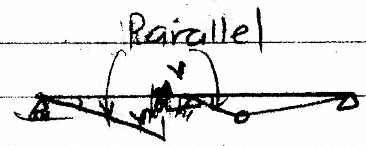
$$= \frac{320}{EI} - \frac{106.67}{EI}$$

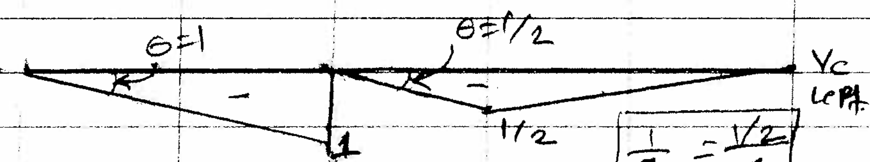
$$\delta_D = \frac{213.33}{EI} \ (\uparrow)$$

Q.3)



Using Muller-Breslau principle

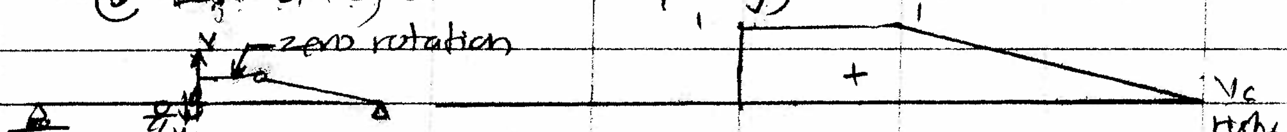




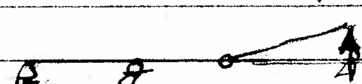
$\frac{1}{8} = \frac{1/2}{4}$

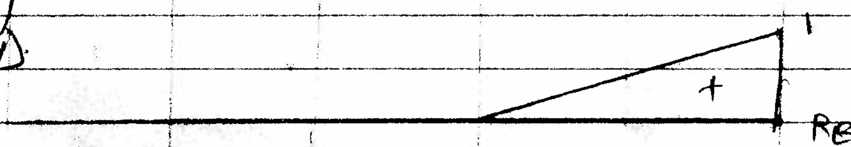
Displaced shape

(Introduce guide roller @ ~~left of~~ 'C') (zero shear capacity)



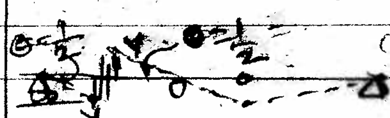
(Introduce guide roller @ right of 'C')
(zero shear capacity).

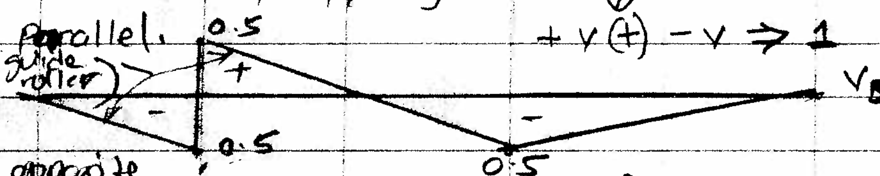




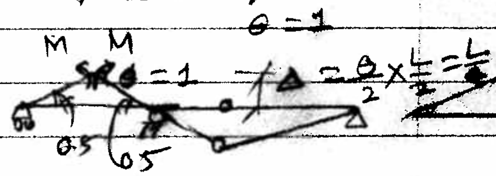
(Introduce vertical reaction @ E with removal of hinge → no vertical support).

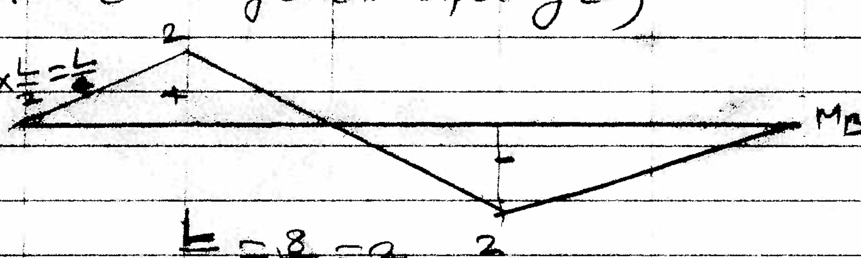
Add. of ord. = load = 1
+V(+) −V ⇒ 1





(Introduce equal & opposite V with guide roller (making shear capacity = 0)





$\frac{L}{4} = \frac{8}{4} = 2$ 2

Introduce equal & opposite moment @ B with hinge formation,
(making moment capacity = 0)