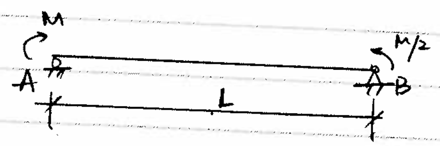
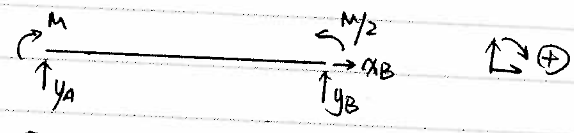
SOLUTION FOR HW #5

P9.3 Compute slope and deflection eqns for the beam, then compute the maximum deflection. Express the answers in terms of M.E.I.L. Hint: Maximum deflection occurs at point of zero slope.



SOLUTION:

1) Compute the external reactions.

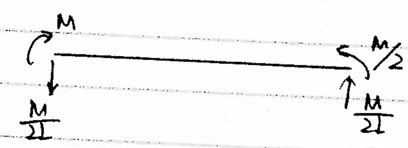


$\Sigma F(x) = 0 \quad x_B = 0$

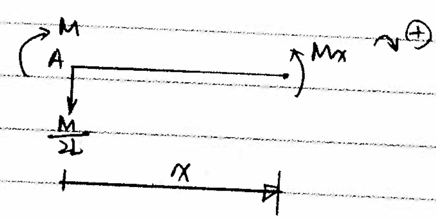
$\Sigma M_B = M + y_A \cdot L - M/2 = 0 \Rightarrow y_A = -\frac{M}{2L}\downarrow$

$\Sigma F_y = y_A + y_B = 0 \quad y_B = \frac{M}{2L}\uparrow$

Draw FBD of this beam:



2) Write expression of M as a function of x. Establish a rectangular coordinate with the origin at support A.



$M - \frac{M}{2L}\cdot x - M_x = 0$

$M_x = M - \frac{M}{2L}\cdot x \ (0 \le x \le L)$

3) Substituting M_x into eqn 9.12 gives:

$$EI\frac{d^2y}{dx^2} = M - \frac{M}{2L}x$$

$$EI\frac{dy}{dx} = Mx - \frac{M}{4L}x^2 + C_1 \quad (1) \quad \text{Ans.}$$

$$EIy = \frac{1}{2}Mx^2 - \frac{M}{12L}x^3 + C_1x + C_2 \quad (2) \quad \text{Ans.}$$

To evaluate the constants of integration C_1 and C_2, we use the boundary conditions at supports A and B, where the deflections are zero.

At A, $x=0$, $y=0$, substituting these values into eqn (2):

$$EI\cdot 0 = 0 + C_2 = 0 \Rightarrow C_2 = 0 \quad \text{Ans.}$$

At B, $x=L$, $y=0$, substituting these values into eqn(2).

$$EI \cdot 0 = \frac{1}{2}ML^2 - \frac{1}{12}ML^2 + C_1 L = 0 \Rightarrow \underline{\underline{C_1 = -\frac{5}{12}ML}} \quad \text{Ans.}$$

Substituting C_1 and C_2 into eqn(2) & eqn(1):

$$EI\frac{dy}{dx} = Mx - \frac{M}{4L}x^2 - \frac{5}{12}ML \qquad \underline{\underline{(3)}} \quad \text{Ans.}$$

$$EIy = \frac{1}{2}Mx^2 - \frac{M}{12L}x^3 - \frac{5}{12}MLx \qquad \underline{\underline{(4)}} \quad \text{Ans.}$$

4) Compute Max deflection:

From problem, we can know the maximum deflection occurs at point of zero slope.

Set eqn(3)=0, we get:

$$\frac{M}{4L}x^2 - Mx + \frac{5}{12}ML = 0$$

$$\Rightarrow x = 3.5L \text{ or } x = 0.472L$$

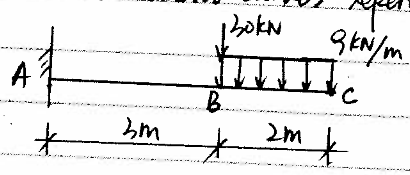
$$\because 0 \le x \le L. \quad \therefore x = 0.472L$$

Substituting $x=0.472L$ into eqn(4), compute maximum deflection.

$$y = \left[\frac{1}{2}M(0.472L)^2 - \frac{M}{12L}(0.472L)^3 - \frac{5}{12}ML(0.472L)\right]/EI$$

$$= \underline{\underline{-\frac{0.094ML^2}{EI}}} \quad \text{Ans.}$$

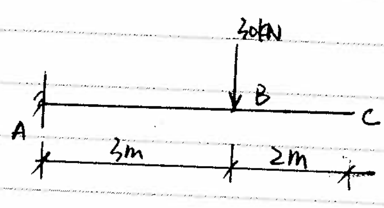
Prob. 9.15 Determine the slope and deflection of point C. Express answers in terms of E, I. Hint: Draw moment curves seperately for the various loadings.



SOLUTION:

The principle of superposition is valid, so we can draw moment curves by applying loads seperately, then add them together.

The total moment curve is made of two parts:



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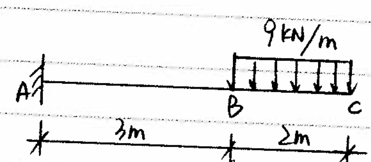
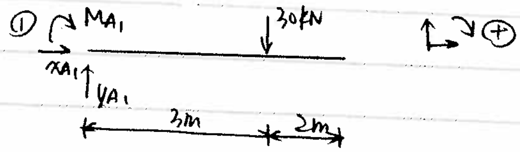


②

1) Compute external reactions of these 2 loading situations seperately.

①

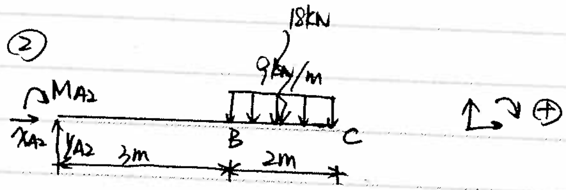


$\Sigma F(x) = x_A = 0$ Ans.

$\Sigma M_A = M_{A1} + 30 \times 3 = 0 \quad M_{A1} = -90\,kN \cdot m$ ↺ Ans.

$\Sigma F(y) = y_{A1} - 30kN = 0 \quad y_{A1} = 30kN \uparrow$ Ans.

②

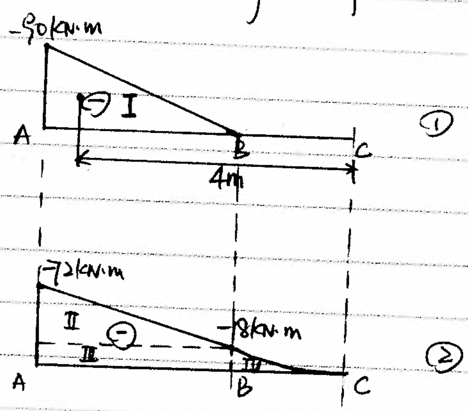


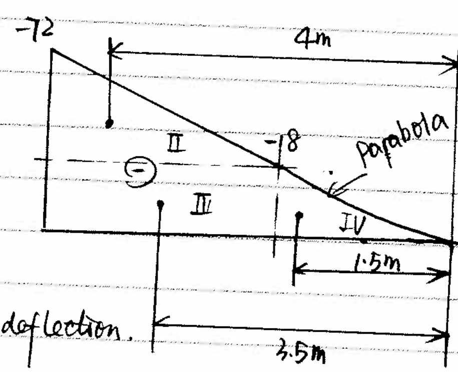
$\Sigma F(x) = x_{A2} = 0$ Ans.

$\Sigma M_A = M_{A2} + 18 \times 4 = 0 \Rightarrow M_{A2} = -72\,kN \cdot m$ ↺ Ans.

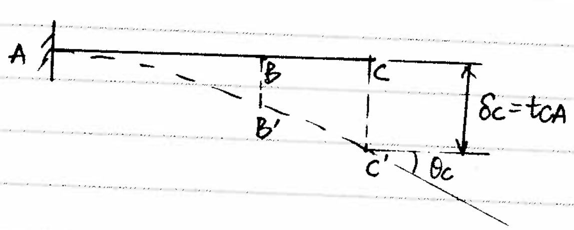
$\Sigma F(y) = y_{A2} - 18 = 0 \quad y_{A2} = 18kN \uparrow$ Ans.

Draw moment diagrams of this beam subjected to these 2 loadings.





2) Draw deflected shape carefully then analyze the deflection.



$\theta_C = \theta_A + \Delta\theta_{AC} = 0 + \Delta\theta_{AC} = \Delta\theta_{AC}$

$= \left[\frac{1}{2}\times 3\times(-90) + \frac{1}{2}\times(-72+18)\times 3 + (-18)\times 3 + \frac{1}{2}\times(-18)\times 2\right] / EI$

$= \underline{\underline{\dfrac{-282}{EI}}}$ Ans.

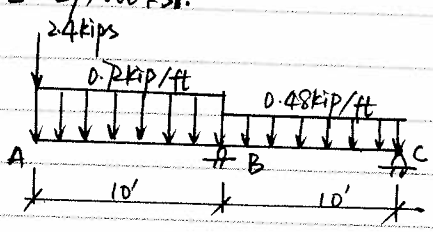
$\delta_C = t_{CA} = \sum_{i=1}^{4} \text{Area} \cdot x$

x should be distance from centroid of area to where we measure deflection

$= \left[\frac{1}{2}\times 3\times(-90)\times 4 + \frac{1}{2}\times 3\times(-54)\times 4 + 3\times(-18)\times 3.5 + \frac{1}{2}\times 2\times(-18)\times 1.5\right] / EI$

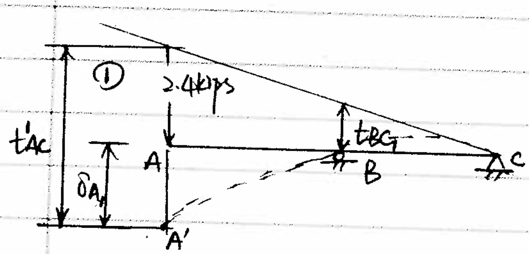
$= \underline{\underline{-1071/EI}}$ Ans.

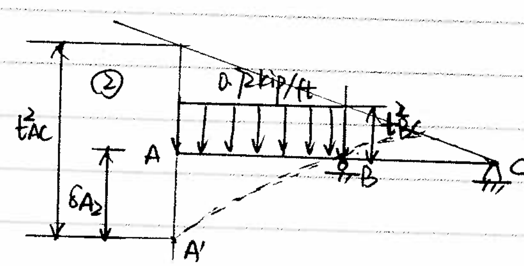
P8.16 The roof beam of a building is subjected to the loading shown below. If a 3/8 inches deflection is permitted at the cantilever end before the ceiling and roofing materials would be damaged, what would the required moment of inertia for the beam.
Use E = 29,000 ksi.

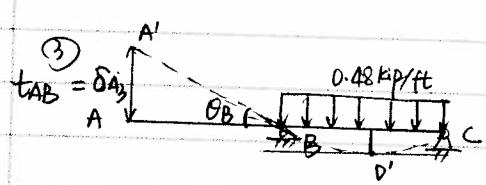


SOLUTION:

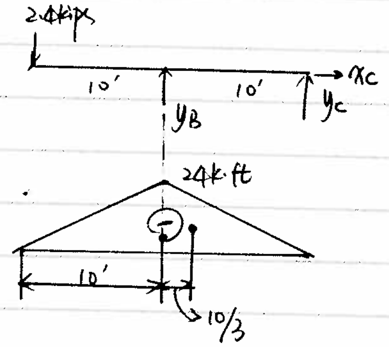
1) This combined loading condition is made up of 3 parts (Use principle of superposition)







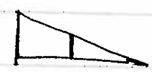
2) Compute external reactions seperately, draw moment diagrams. (+)

① 

$\Sigma F(x) = x_C = 0$

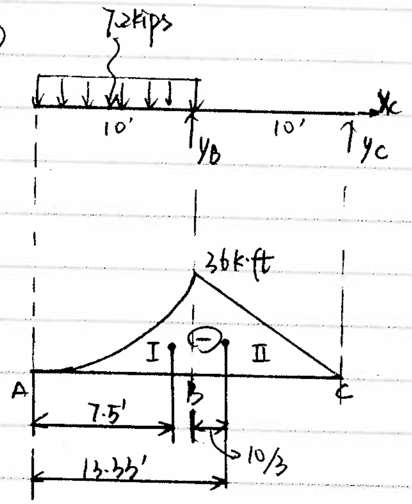
$\Sigma M_B = -2.4\times10 + y_C\times10 = 0 \Rightarrow y_C = -2.4\text{kips}\downarrow$ Ans.

$\Sigma F_y = -2.4 + y_B - 2.4 = 0 \Rightarrow y_B = 4.8\text{kips}\uparrow$ Ans.

$\delta_{A1} = t_{AC_1} - 2t_{BC_1}$ 

$= \left[\frac{1}{2}\times20\times24\times10 - 2\times\left(\frac{1}{2}\times24\times10\times10/3\right)\right]/EI$

$= -\frac{2400}{EI} - \left(-\frac{800}{EL}\right) = -\frac{1600}{EI}$ Ans.

② 

$\Sigma F(x) = x_C = 0$

$\Sigma M_B = -7.2\times5 - y_C\times10 = 0 \Rightarrow y_C = -3.6\text{kips}\downarrow$ Ans.

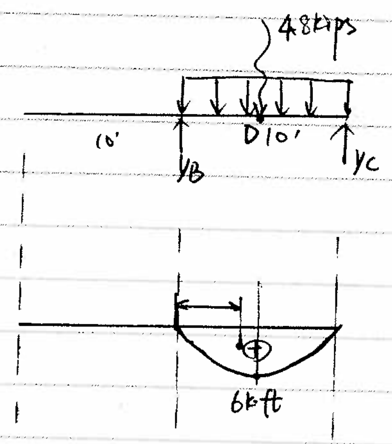
$\Sigma F_y = -7.2 - 3.6 + y_B = 0 \Rightarrow y_B = 10.8\text{kips}\uparrow$ Ans.

$\delta_{A2} = t_{AC_2} - 2t_{BC_2}$

$= \left[\frac{1}{3}\times10\times(-36)\times7.5 + \frac{1}{2}\times10\times(-36)\times13.33\right]/EI$

$-\left(-2\times\frac{1}{2}\times36\times10\times\frac{10}{3}\right)/EI$

$= -\frac{3300}{EI} - \left(-\frac{1200}{EI}\right) = -\frac{2100}{EI}$ Ans.

③ 

Under symmetrical loading, so reactions must be symmetrical, $y_B = y_C = 2.4\text{kips}\uparrow$ Ans.

$\delta_{A3} = \theta_B\times(10')$

$\theta_B = \theta_D + \Delta_{DB} = \Delta_{DB} = \frac{2}{3}\times5\times(6)/EI = \frac{20}{EI}$

$\therefore \delta_{A3} = \frac{200}{EI}$ Ans.

Total deflection $\Delta A = \delta_{A1} + \delta_{A2} + \delta_{A3}$

$= \frac{-1600}{EI} - \frac{2100}{EI} + \frac{200}{EI} = -\frac{3500}{EI}$ Ans.

3) Compute I.

For $\Delta_{max} = 3/8$ in , $E = 29{,}000$ ksi

$$I_{(req)}' = \frac{3500 \times 1728\ in^3/ft^3}{29{,}000\ k/in^2 \times 3/8\ in} = \underline{\underline{556\ in^4}}$$ Ans.