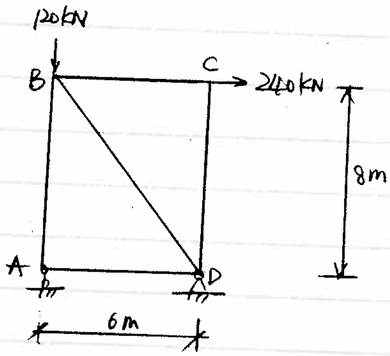
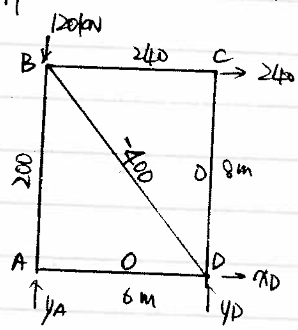
HW #6 SOLUTIONS:

Pro.4 For the truss in Fig shown below, compute the vertical displacement of joint B and the horizontal displacement of roller at joint A.

All bars: $A = 2{,}500\ mm^2$, $E = 200GPa$.



1) Compute external reactions and bars forces of Fp system: ↑ ↻ ⊕



$\Sigma F(x) = x_D + 240 = 0 \quad x_D = -240kN (\leftarrow)$

$\Sigma M_D = y_A \times 6 - 120 \times 6 + 240 \times 8 = 0 \quad y_A = -200kN (\downarrow)$

$\Sigma F_y = -120 + y_A + y_D = 0 \Rightarrow y_D = 120 - y_A = 320kN (\uparrow)$

Bar forces (Joint Method) Assuming all bar forces to be tension.

A ↑ F_{AB}, ↓ 200kN $F_{AB} - 200kN = 0$

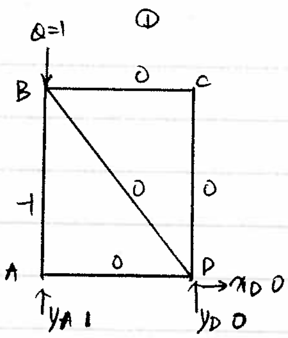
$F_{AB} = 200kN (T)$

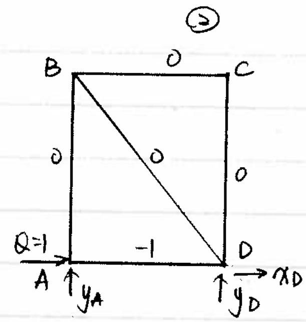
C F_{CB} → 240kN, ↓ F_{CD} $F_{CD} = 0$, $F_{CB} = 240kN$

B ↓ 120kN, → 240kN, ↓ 200kN, ↘ F_{BD}. $F_{BD} = -\sqrt{240^2 + 320^2} = -400kN$

2) u system.

To compute the vertical displacement of joint B and horizontal displacement of roller A, we apply dummy loads of 1 kN ~~horiz~~ vertically at joint B, or horizontally at roller A, respectively. Thus, we have 2 Fa systems shown below.





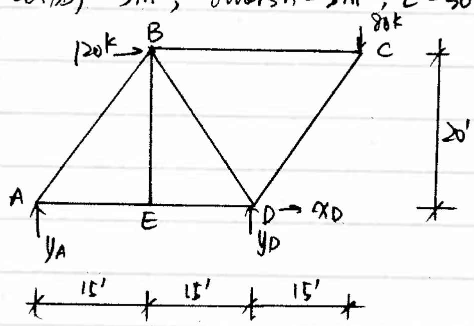
We can compute the external reactions and bar forces for these 2 Q systems.

$\because \Sigma Q\delta_P = \Sigma F_Q \frac{F_P L}{AE}$

For ①: $1 \cdot \delta_{BV} = \frac{-1\times 200\times 8\times 10^3}{200\times 10^9\times 2500\times 10^{-6}} = \underline{\underline{-3.2\text{mm}}}\ (\uparrow)$ Ans.

For ② $1\cdot\delta_{AX} = \frac{-1\times 0}{200\times 10^9\times(2500\times 10^{-6})} = \underline{\underline{0}}$ Ans.

Prob5. Compute the vertical and horizontal components of displacement of joint E produced by the loads
$A(AB, CD, BD) = 5in^2$; others $A = 3in^2$, $E = 30,000 kips/in^2$.

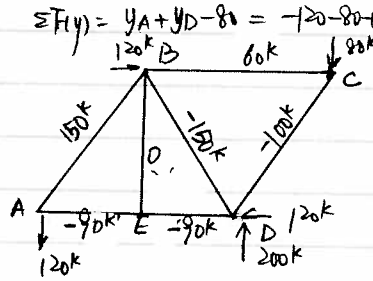


1) Compute external reactions & all bar forces of P system. ⊕

$\Sigma F(x) = 120 + x_D = 0 \quad x_D = -120\text{ kips } (\leftarrow)$

$\Sigma M_D = y_A\times 30 + 120\times 20 + 80\times 15 = 0 \Rightarrow y_A = -120\text{ kips } (\downarrow)$

$\Sigma F(y) = y_A + y_D - 80 = -120 - 80 + y_D = 0 \Rightarrow y_D = 200\text{ kips } (\uparrow)$



Compute bar forces using Method of joints.

Ⓐ

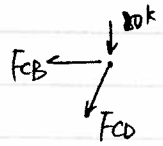
$\Sigma F_y = F_{ABy} - 120 = 0 \Rightarrow F_{ABy} = 120^k$

$F_{AB} = \frac{5}{4}\times 120^k = 150^k (T)$,

$\Sigma F(x) = F_{AE} + F_{ABx} = 0 \quad F_{AE} = -F_{ABx} = -F_{AB}\cdot\frac{3}{5} = -150\times 3/5 = -90^k (C)$

(E) $F_{EA} \longleftarrow \longrightarrow F_{ED}$ $\quad F_{EA} = F_{ED} = -90^k$

(C)

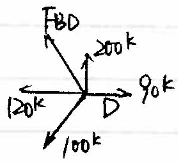


$\Sigma F_{(y)} = -80 - F_{CDy} = 0 \quad F_{CDy} = -80\text{ kips},\ F_{CD} = \frac{5}{4} F_{CDy} = \frac{5}{4} \times (-80) = -100\text{ kips (C)}$

$\Sigma F_{(x)} = -F_{CB} - F_{CDx} = 0$

$\Rightarrow F_{CB} = -F_{CDx} = -F_{CDy} \times \frac{3}{4} = -(-80) \times \frac{3}{4} = 60\text{ kips (T)}$

(D)

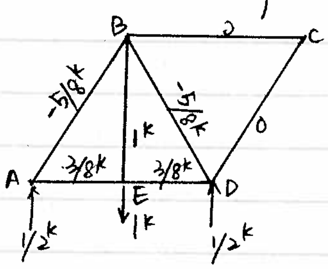


$\Sigma F_{(x)} = -F_{BDx} - 120 + 90 - 100 \times \frac{3}{5} = 0$

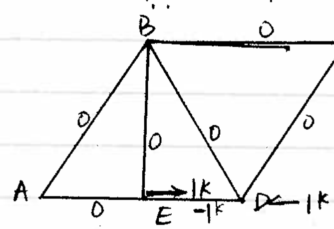
$F_{BDx} = -90\text{ kips} \quad F_{BD} = -90 \times \frac{5}{3} = -150\text{ kips (C)}$

2) To compute the displacement of joint E, we apply dummy loads of 1 kip vertically and horizontally at joint E (1 at a time).

Thus, we have 2 Q system shown below. Then compute bar forces of Q system.



① ⇑ Q_1-system for δ_{EV}



② ⇑ Q_2-system for δ_{EH}

3) Compute displacement

$$1^k \delta_{EH} = \Sigma F_Q \frac{F_P L}{AE}$$

$$= \frac{-90 \times (-1) \times (15' \times 12)}{3 \times (30,000)} = \underline{\underline{0.18''}}\ (\rightarrow) \text{ Ans.}$$

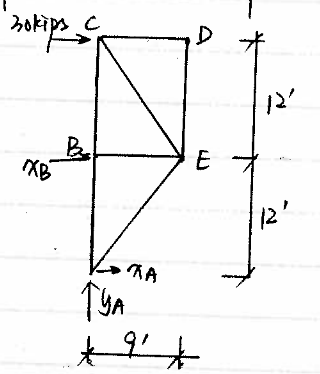
$$1^k \delta_{EV} = \Sigma F_Q \frac{F_P \cdot L}{AE}$$

$$= (\quad)_{AB} + (\quad)_{AE} + (\quad)_{BE} + (\quad)_{DE} + (\quad)_{BD} + (\quad)_{BC} + (\quad)_{CD}$$

$$= \frac{150 \times (-5/8) \times (25 \times 12)}{5 \times 30,000} + \frac{(-90) \times (3/8) \times 15 \times 12}{3 \times 30,000} + 0 + \frac{(-90) \times (3/8) \times 15 \times 12}{3 \times 30,000} + \frac{(-150) \times (-5/8) \times 25 \times 12}{5 \times 30,000} + 0$$

$$= \underline{\underline{-0.135''\ (\uparrow)}} \text{ Ans.}$$

Prob 7. Compute the vertical deflection of joint D produced by 30 kips load ↓: $A = 2in^2$, $E = 9000 \text{ kips}/in^2$.



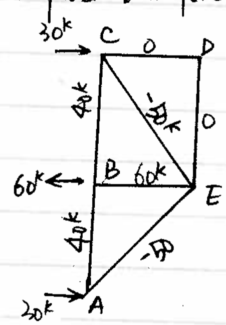
1) Compute bar forces and external reactions of Fp system.

$\Sigma F(y) = y_A = 0$

$\Sigma M(A) = 30 \times 24 + x_B \times 12 = 0 \quad x_B = -60 \text{ kips} (\leftarrow)$

$\Sigma F(x) = 30 + x_B + X_A = 30 - 60 + x_A = 0 \quad x_A = 30 \text{ kips} (\rightarrow)$

Compute bar forces, using Method of joint.



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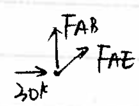
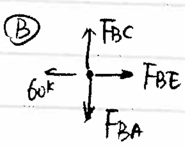


$\Sigma F(x) = 30 + F_{AEx} = 0 \quad F_{AEx} = -30^k, \; F_{AE} = -30 \times \frac{5}{3} = -50^k (C)$

$\Sigma F(y) = F_{AB} + F_{AEy} = 0 \Rightarrow F_{AB} = 40^k (T)$

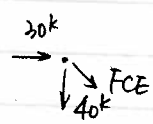
Ⓑ



$F_{BC} = F_{BA} = 40^k$

$F_{BE} = 60^k$

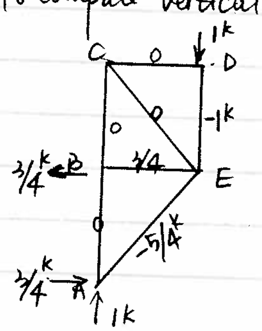
Ⓒ



$\Sigma F(x) = 30^k + F_{CEx} = 0 \quad F_{CEx} = -30^k$

$F_{CE} = -30 \times \frac{5}{3} = -50^k (C)$

2) To compute vertical deflection of joint D, apply a dummy force of 1 kips vertically at D.



3) Compute displacement.

$$1^k \, \delta_{DV} = \frac{60^k \times \frac{3}{4}k \times (9 \times 12)}{2 \times 9000} + \frac{-50 \times (-5/4) \times (15 \times 12)}{2 \times 9000}$$

$$= \underline{\underline{0.895 in (\downarrow)}}$$

Ans.