

NAME: SOLUTION RPI ID #: _____

ENGR 2600 Modeling and Analysis of Uncertainty Section 3 Fall 2008 Exam 1

Rules of the Game:

1. Work entirely alone. Do not give or solicit assistance from any other student.
2. Do not open any IM or email programs or web browsers. Turn off cell phones. Running such software will be regarded as evidence of cheating.
3. This exam is open-book/open-notes/open-computer. Use any written materials you wish.
4. There are 5 questions, weighted equally. Answer all 5. Tip: Do the easier questions first.
5. You must show your work to receive credit, even if the number you write down is correct.
6. Feel free to use the restrooms as necessary.
7. If you have a question, bring it down front so as to minimize disruption.

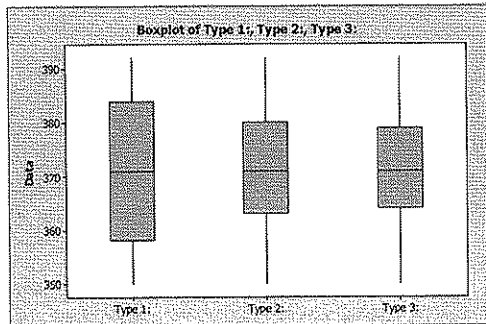
You've got to fight
For your right
To paaaarty!

-- Beastie Boys

Question 1. Read in the Devore text dataset **ex01-75.MTW**. You can read about this dataset on page 43 of the text.

Create side-by-side boxplots of the fatigue limits of the 3 types of rope wire. Answer Yes or No:

- a. YES The levels look about the same.
- b. NO The spreads look about the same.
- c. YES The overall shapes look about the same.



Name and report the sample values of 3 measures of level for Type 1:

- d. Name MEAN Sample value 370.80
- e. Name MEDIAN Sample value 371.00
- f. Name TRIMMED MEAN Sample value 370.77

Name and report the sample values of 3 measures of spread (expressed in MPa) for Type 1:

- g. Name STD. DEV. Sample value 14.41
- h. Name IQR Sample value 26.00
- i. Name RANGE Sample value 42.00

Descriptive Statistics: Type 1, Type 2, Type 3:

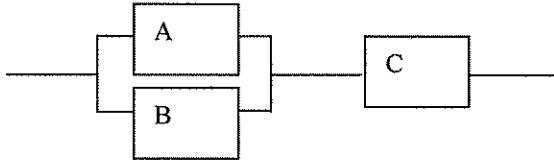
Variable	N	Mean	TrMean	StDev	Minimum	Median	Maximum	Range	IQR
Type 1:	15	<u>370.80</u>	<u>370.77</u>	<u>14.41</u>	350.00	<u>371.00</u>	392.00	<u>42.00</u>	<u>26.00</u>
Type 2:	15	371.00	371.00	11.89	350.00	371.00	392.00	42.00	17.00
Type 3:	15	371.00	371.00	10.54	350.00	371.00	392.00	42.00	15.00

Calculate the lowest Type 3 fatigue limit that would be considered a high outlier.

- j. Lower limit for high outliers 423

$$\text{High} > Q_3 + 1.5 \text{ IQR} = 384 + 1.5 \times 26 = 423$$

Question 2. Consider the following 2-stage system S, which has subsystems A and B in parallel in the first stage and subsystem C in the second stage. Denote the probabilities of successful operation as $P(S)$, $P(A)$, $P(B)$ and $P(C)$.

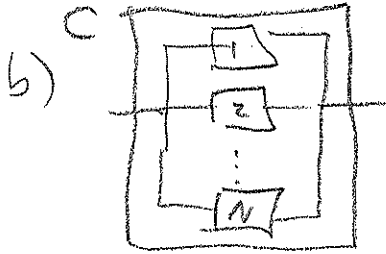


- Write $P(S)$ as a function of $P(A)$, $P(B)$, and $P(C)$. Assume failures are independent.
- Suppose subsystem C is itself composed of N parallel components, each with probability p of working. Write the expression for $P(C)$ as a function of N and p .

$$a) P(S) = P(A \cup B) \cdot P(C)$$

$$P(A \cup B) = 1 - P(A') \cdot P(B') = 1 - [1 - P(A)] [1 - P(B)]$$

$$\therefore P(S) = \{ 1 - [1 - P(A)] [1 - P(B)] \} P(C)$$



$$P(C) = 1 - P(C')$$

$$P(C') = P(1') \cdot P(2') \cdots P(N')$$

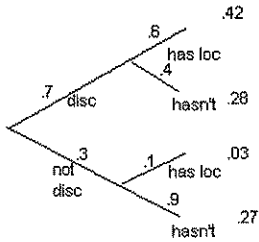
$$= (1 - p)^N$$

$$\therefore P(C) = 1 - (1 - p)^N$$

Question 3. Answer Devore's exercise 2.4-60, reproduced here for your convenience.

Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.

- If it has an emergency locator, what is the probability that it will not be discovered? [Show your work.]
- If it does not have an emergency locator, what is the probability that it will be discovered? [Show work.]



$$a. P(\text{not disc} | \text{has loc}) = \frac{P(\text{not disc} \cap \text{has loc})}{P(\text{has loc})} = \frac{.03}{.03 + .42} = .067 = (a)$$

$$b. P(\text{disc} | \text{no loc}) = \frac{P(\text{disc} \cap \text{no loc})}{P(\text{no loc})} = \frac{.28}{.55} = .509 = (b)$$

Question 4. Show your work.

- Random variable X has a binomial distribution with parameters $n = 2$ and $p = 0.2$. What is $P(X = 0)$?
- For the same X , compute $E[e^{-X}]$.
- Random variable Y has a Poisson distribution with parameter $\lambda = 3.3$. What is $P(Y > 2)$?
- A record of the number of accidents per month at a certain intersection over 36 months has a sample mean of 0.81 and a sample standard deviation of 5.3. Do these summary statistics support the use of a Poisson model for the number of accidents per month? Explain your reasoning.
- A spacecraft is designed to collect small particles that collide with its special aerogel plate. It is expected that the number of particles collected in a single day will have a Poisson distribution with mean 0.01. If the mission lasts for 1,000 days, what is the probability that it will collect at least 10 particles?

$$a) P(X=0) = \binom{2}{0} \cdot 0.2^0 \cdot (1-0.2)^2 = 0.64$$

$$b) E[e^{-X}] = \sum_{x=0}^2 e^{-x} p(x) = e^{-0} p(0) + e^{-1} p(1) + e^{-2} p(2) \\ = 1 \times 0.64 + 0.368 \times 0.32 + 0.135 \times 0.04 = 0.763135$$

$$c) P(Y > 2 | \lambda = 3.3) = 1 - P(Y \leq 2 | \lambda = 3.3) = 1 - 0.359426 = 0.640574$$

d) For: Poisson(λ), $E[X] = \lambda$, $SD[X] = \sqrt{\lambda}$
Estimate λ by $\bar{x} = 0.81$ so expect $s \approx \sqrt{0.81} = 0.9$.
The observed std. deviation is way too big, so NO, not Poisson.

e) Poisson for 1,000 days \sim Poisson with mean = $1000 \times 0.01 = 10$.

$$P(\text{"at least 10"} | \lambda = 10) = 1 - P(\leq 9 | \lambda = 10) = 1 - 0.457930 = 0.54207$$

Question 5. Show your work.

- 3 points → a) Random variable X has an exponential distribution with parameter λ . Express the 90th percentile of the distribution of X as a function of λ .
- b) Random variable Y has a normal distribution with parameters μ and σ . Express the 15th percentile of the distribution of Y as a function of μ and σ .
- 4 points → c) Your company produces small glass spheres. Your production process is a bit loose, so the radius of the spheres varies. The distribution of radius is normal with mean 100 nm and standard deviation 5 nm. A potential customer requires that the spheres have volume ($= 4/3\pi r^3$) $< 4.3 \times 10^6$ cubic nanometers. Determine the fraction of your production that meets this customer's specification.

$$a) P(X \leq x_0) = 1 - e^{-\lambda x_0} = 0.9 \Rightarrow x_0 = -\frac{\ln(0.1)}{\lambda} = \frac{2.303}{\lambda}$$

$$b) 15^{\text{th}} \text{ qtile of } Z \sim N(0,1) = -1.03643$$

so general answer is $\mu - 1.03643\sigma$

$$c) P(\text{volume} \leq 4.3 \times 10^6) = P\left(\frac{4}{3}\pi r^3 \leq 4.3 \times 10^6\right) = P\left(r \leq \sqrt[3]{\frac{3}{4\pi} \times 4.3 \times 10^6}\right)$$
$$= P(r \leq 100.8773)$$
$$= 0.569641$$