NAME: SOLUTIONS RPI ID #: ____________________

ENGR 2600 Modeling and Analysis of Uncertainty Section 3 Fall 2008 Exam 2

Rules of the Game:

1. Work entirely alone. Do not give or solicit assistance from any other student.
2. Do not open any IM or email programs or web browsers. Turn off cell phones. Running such software will be regarded as evidence of cheating.
3. This exam is open-book/open-notes/open-computer. Use any written materials you wish.
4. There are 5 questions, weighted equally. Answer all 5. Tip: Do the easier questions first.
5. You must show your work to receive credit, even if the number you write down is correct.
6. Feel free to use the restrooms as necessary.
7. If you have a question, bring it down front so as to minimize disruption.

We don't need no education.
We don't need no thought control.
No dark sarcasm in the classroom.
Teacher, leave them kids alone.
Hey, teacher, leave them kids alone!

-- *Pink Floyd "The Wall, Pt.2"*

Question 1. Short answer.

a) Central Limit The theorem proving that the distribution of the sum of large numbers of i.i.d. random variables tends toward the normal distribution.

b) Bias The difference between the mean value of a statistic and the actual value of the parameter estimated by that statistic.

c) Standard Error The standard deviation of the sampling value of a statistic.

d) Correlation A dimensionless measure of the association between two random variables.

e) Random Sample A collection of i.i.d. random variables.

Question 2. Consider the following joint distribution of X and Y.

		Y: 6	Y: 12	Y: 24	row sum
X	0	0.07	0.02	0.11	0.20
X	1	0.10	0.12	0.07	0.29
X	2	0.01	0.01	0.11	0.13
X	3	0.10	0.12	0.01	0.23
X	4	0.12	0.02	0.01	0.15
col sum		0.40	0.29	0.31	1.00

a) Write the marginal distribution of X.

These are the row sums: $\{0.20, 0.29, 0.13, 0.23, 0.15\}$

b) Compute E[1/Y].

$E[1/Y] = \frac{1}{6} \times 0.40 + \frac{1}{12} \times 0.29 + \frac{1}{24} \times 0.31 = 0.10375$

c) Write the conditional distribution $f_{Y|X}(y|x=0)$.

This is the top row: $\{0.07, 0.02, 0.11\}/0.20$
normalized
$= \{0.35, 0.10, 0.55\}$

d) Compute P[X+Y=16].

One way to get this $X=4, Y=12$

so $P(X+Y=16) = P(X=4 \cap Y=12) = 0.02$

e) Are X and Y independent? Prove your conclusion using any cell as an example.

No. Example $P(X=4 \cap Y=6) = 0.12$

but $P(X=4) \cdot P(Y=6) = 0.15 \times 0.40 = 0.06 \neq 0.12$

Question 3. Suppose you manufacture a miniature box with random dimensions. The 3 dimensions (length, height and width) have a multivariate normal distribution with the following means and covariances:

Means

	Length	Height	Width
mean	50	10	25

Covariances

	Length	Height	Width
Length	9	1.5	4.2
Height	1.5	1	-1
Width	4.2	-1	4

a) Determine the standard deviation of Height.

$$\sigma_H = \sqrt{1} = 1$$

b) Rewrite the covariance matrix above for the case in which all the variables are uncorrelated.

$$\begin{bmatrix} 9 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

c) Determine the correlation between Height and Width.

$$\rho_{HW} = \frac{Cov[H, W]}{\sqrt{V[H] \cdot V[W]}} = \frac{-1}{\sqrt{1 \times 4}} = -0.5$$

d) Determine P(Width < 26).

$$P(W < 26) = P\left(Z < \frac{26-25}{2}\right) = P\left(Z < \tfrac{1}{2}\right) = 0.691462$$

e) Determine the variance of Length+Height+Width.

$V[L+H+W]$ = sum of all terms in covariance matrix

$= 23.4$

Question 4. Read in the Minitab-supplied dataset *Exh_stat.MTW*. Focus on variable C10, EnzymeActivity.

a) Report the standard error of the sample mean and determine the sample size needed to reduce the standard error below 0.05.

$SE = 0.111807 = \frac{s}{\sqrt{n}} = \frac{0.387310}{\sqrt{12}}$

To get $SE < 0.05 \Rightarrow \frac{s}{\sqrt{n}} < 0.05 \Rightarrow n > \left(\frac{s}{0.05}\right)^2 = \left(\frac{0.387}{0.05}\right)^2 = 59.9076$

So need $n = 60$

b) Determine the 95% CI for the mean.

From Minitab $(0.268915, 0.761085)$

i.e. $\bar{x} \pm t_{n-1, \alpha/2} \frac{s}{\sqrt{n}} = 0.515 \pm 2.20 \cdot \frac{0.387}{\sqrt{12}}$

c) Report the value of the sample standard deviation and estimate its standard error using at least 20 bootstrap replications. Copy down the Minitab commands (script) you used.

$SE^*(s) = 0.062$ using $B = 1{,}000$ replications

script:

```
Sample 12 'EnzymeActivity' 'bootsample';
    Replace.
StDev 'bootsample' k1.
stack k1 C3 C3.
```

Question 5. A simple electrical circuit consists of 3 resistors connected in parallel. The resistors have values of R1, R2 and R3 ohms. This circuit has resistance equivalent to a single resistor of Req = 1/((1/R1)+(1/R2)+(1/R3)) ohms. The 3 resistances are supposed to all be 100 ohms, but manufacturing variability means that the actual resistance has a normal distribution with mean 100 and standard deviation 5. Assume the resistance values are independent. Use Monte Carlo simulation row-wise (i.e., no need for scripts) with 1,000 replications to answer these questions:

a) Estimate the mean of Req.

$\bar{X} = 33.266$ or 33.228 in 2 replications

b) Report the standard error of the estimate of the mean.

$SE(\bar{x}) = 0.0305$ or 0.0307

c) Estimate the standard deviation of Req.

$S = 0.965$ or 0.969

d) Estimate the 5th percentile of Req.

31.7484 using Stat > Tables > Tally...
or 31.6854

e) Is the distribution of Req well-modeled as normal? Justify your answer with reference to the appropriate plot.

Yes, because a normal probability plot is quite linear.