Section ____________

Your Name SOLUTION
(Print Clearly)

RPI ID# ____________

ENGR-2600 Modeling & Analysis of Uncertainty

Fall 2008

Exam #2

Laptop (with MINITAB) required

Please answer any five (5) of the six questions
Each question is worth 10 points

1. Open Book/open notes. Use any written aids you wish.
2. Write all answers on this test. Be sure to put your name on it, ID number and the section number.
3. Show all your work – you may get partial credit, if necessary.
4. Work entirely alone. Do not give or solicit assistance from any other student.
5. Do not open any IM or email programs or web browsers. Turn off cell phones.

Please indicate which problem you do NOT want graded____________.

SOLUTION

Problem 1: (Linear Combinations)

Consider three independent normal random variables $X_1 \sim N(30, 3^2)$; $X_2 \sim N(15, 2^2)$; $X_3 \sim N(20, 1^2)$ and let

$$Y = 3X_1 - 2X_2 - X_3$$

Without using MINITAB or simulation, find analytically,

a. $E(Y) = 3E(X_1) - 2E(X_2) - E(X_3) = 3(30) - 2(15) - 20 = 40$

b. $V(Y) = 3^2 V(X_1) + 2^2 V(X_2) + V(X_3) = 3^2(9) + 2^2(4) + 1 = 98$

c. Probability Distribution of Y - Normal

d. $P[Y > 50] = P\left[Z > \frac{50-40}{\sqrt{98}}\right] = P[Z > 1.010]$

$= 1 - \Phi(1.010)$

$= 1 - 0.8437$

$= 0.1563$

Problem 2: (Standard Error)

Samples of size n = 16 are drawn from a normal population

$$X \sim N(50, 4^2)$$

Evaluate

a. $P[42 < X < 58] = P\left[\frac{42-50}{4} < Z < \frac{58-50}{4}\right] = P[-2 < Z < +2] = .9545($

b. $\mu_{\bar{x}} = E(\bar{x}) = \mu = 50$

c. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{16}} = \frac{4}{4} = 1$

d. $P[48 < \bar{x} < 52] = P\left[\frac{48-50}{1} < Z < \frac{52-50}{1}\right] = P[-2 < Z < 2]$

$= .97725 - .02275$

$= 0.95450$

e. $P[\bar{x} > 47]$

(e) $P[\bar{x} > 47] = P\left[Z > \frac{47-50}{1}\right] = P[Z > -3]$

$= \Phi(3) = 0.99865$

~~= 0.00135~~

Problem 3:

Please fill in the blanks in the following:

a. Samples of size n = 40 are drawn from a population with an exponential distribution having a mean = 5. Then the sampling distribution of sample means $\bar{x}$ is approximately Normal because of Central Limit Theorem (CLT)

b. To check the normality, we use a Normal Probability plot.

c. If $X \sim N(20, 2^2)$ and $Y \sim N(15, 1^2)$ and $W = 3X - 2Y$, then $E(W) = 60 - 30 = 30$; $V(W) = 3^2(2^2) + 2^2(1) = 40$ and distribution of W is Normal.

d. The standard error of $\bar{x}$ from a population $X \sim N(50, 9^2)$ is $\sigma_{\bar{x}} = 0.36$; then the sample size; n = 625.

e. If $\sigma_x = .747$, $\sigma_y = .647$ and Cov (X, Y) = 0.347, then the correlation coefficient, $\rho_{x,y} =$ .7180.

f. If X and Y are independent, $P[X < a, Y < b] = P[X < a] * P[Y < b]$

g. Of the sample mean $\bar{x}$ and sample median $\tilde{x}$ for a normally distributed data set, which is a better point estimator of population mean μ? Sample Mean $\bar{x}$.

h. What MINITAB command would you employ to check if the given data conform to a hypothesized normal distribution? Graph > Prob Plot > Normal

____________.

i. If X and Y are independent random variables, then $\rho_{x,y} =$ 0.

j. If p (X,Y) is the joint probability mass function, then $\sum_y p(x, y) =$ marginal pmf

Problem 4: (Simulation)

An experiment yields the nominal values and associated uncertainties of variables X, Y and Z so that we can assume

$$X \sim N(25, 1^2)$$
$$Y \sim N(5.0, 0.3^2)$$
$$Z \sim N(3.5, 0.2^2)$$

a. Find the nominal value and incertainty of $W = X \, \ell n\,(Z^2 + Y)$

Mean of $W = 70.908$; $\sigma_W = 3.528$ $\qquad W \sim N(70.91, 3.53^2)$

b. Is the probability distribution of W normal?

Yes – Normal

Problem 5: (Joint Probability Distributions)

In a randomly chosen lot of 100 bolts, let X be the number that fail to meet a length specification, and let Y be the number that fail to meet a diameter specification. Suppose the joint pmf p(X, Y) of X and Y is given in the following table:

X \ Y	0	1	2
0	.40	.12	.08
1	.15	.08	.03
2	.10	.03	.01

a. Show the marginal pmf's:

X	0	1	2
$p_X(x)$	.60	.26	.14

Y	0	1	2
$p_Y(y)$	.65	.23	.12

b. Compute

i. $\mu_X =$ $0(.60) + 1(.26) + 2(.14) = .54$

ii. $\mu_y =$ $0(.65) + 1(.23) + 2(.12) = .47$

iii. $E(XY) =$ $1(.08) + 2(.03) + 2(.0\cancel{8}3) + 4(.01) = \cancel{.24}$ $\cancel{0.3}$ 0.24

iv. $\text{Cov}(X, Y) =$ $E(XY) - \mu_X \mu_Y = \cancel{.24}\ 0.24 - (.57)(.47)$

$= \cancel{.24}\ 0.34 - .268$

$= \cancel{-.028}$

$\cancel{0.032}$

Problem 6: (Simulation)

Using MINITAB, consider the generation of 500 samples of size n = 9 each from an exponential population with mean = 5. Consider the standard deriation s of each sample.

a. Estimate E (s) = 4.54

b. Is the sampling distribution of s normal? No

c. Estimate the Standard Deviation of s, σ_s = 1.93

d. Is the sampling distribution of s exponential? No

Describe the steps and MINITAB commands in your procedure. Justify your answers to (b) and (d) by referring to appropriate graphs.

(b) Normal Prob. Plot

(d) Exponential Prob. Plot